The Fast Fourier Transform (FFT) is a fast method for evaluating the complex exponential sum,

$$X_n = \sum_{j=0}^{N-1} x_k W^{nk} \quad n = 0 \ldots N-1$$

in $O(n\, lg\, n)$ operations where $W = e^{-2\pi i/N}$ is an nth root of unity and the symbol i denotes $\sqrt{-1}$ and N is a power of 2: $N = 2^g$.

Express the summation indices in binary,

$$n = 2^{g-1} n_{g-1} + 2^{g-2} n_{g-2} + \ldots + n_0$$

$$k = 2^{g-1} k_{g-1} + 2^{g-2} k_{g-2} + \ldots + k_0$$

to expand the sum into

$$X(n_{g-1}, n_{g-2}, \ldots, n_0) = \sum_{k_0=0}^{1} \sum_{k_1=0}^{1} \cdots \sum_{k_{g-1}=0}^{1} \quad x(k_{g-1}, k_{g-2}, \ldots, k_0) W^{nk}$$

Expand out the complex exponential term

$$\begin{aligned} W^{nk} = W^{(2^{g-1} n_{g-1} + \ldots + n_0)(2^{g-1} k_{g-1})} \times \\ W^{(2^{g-1} n_{g-1} + \ldots + n_0)(2^{g-2} k_{g-2})} \times \ldots \times \\ W^{(2^{g-1} n_{g-1} + \ldots + n_0)(k_0)} \end{aligned}$$

Simplify using

$$W^{(m2^g)} = W^{mN} = (W^N)^m = 1^m = 1$$

using

$$W^N = (e^{2\pi i/N})^N = e^{2\pi i} = 1$$

which gives us,

$$W^{nk} = W^{(n_0)(2^{g-1} k_{g-1})} W^{(2n_1 + n_0)(2^{g-2} k_{g-2})} \ldots W^{(2^{g-1} n_{g-1} + \ldots + n_0)(k_0)}$$

Substitute back into the sum to give

$$\sum_{k_0=0}^{1} \sum_{k_1=0}^{1} \cdots \sum_{k_{g-1}=0}^{1} \quad x(k_{g-1}, k_{g-2}, \ldots, k_0) W^{(n_0)(2^{g-1} k_{g-1})} W^{(2n_1 + n_0)(2^{g-2} k_{g-2})} W^{(2^{g-1} n_{g-1} + \ldots + n_0)(k_0)}$$