Finite Fourier Transform of a Sine Wave with Windowing

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Abstract

We also show that windowing the DFT to force periodicity greatly reduces the Gibbs phenomenon.

1. Digitized Sine Wave of Known Frequency

Construct a digitized sine wave on n = 512 sample points with sampling frequency fs = 100 Hz, frequency f = 1.2 Hz,

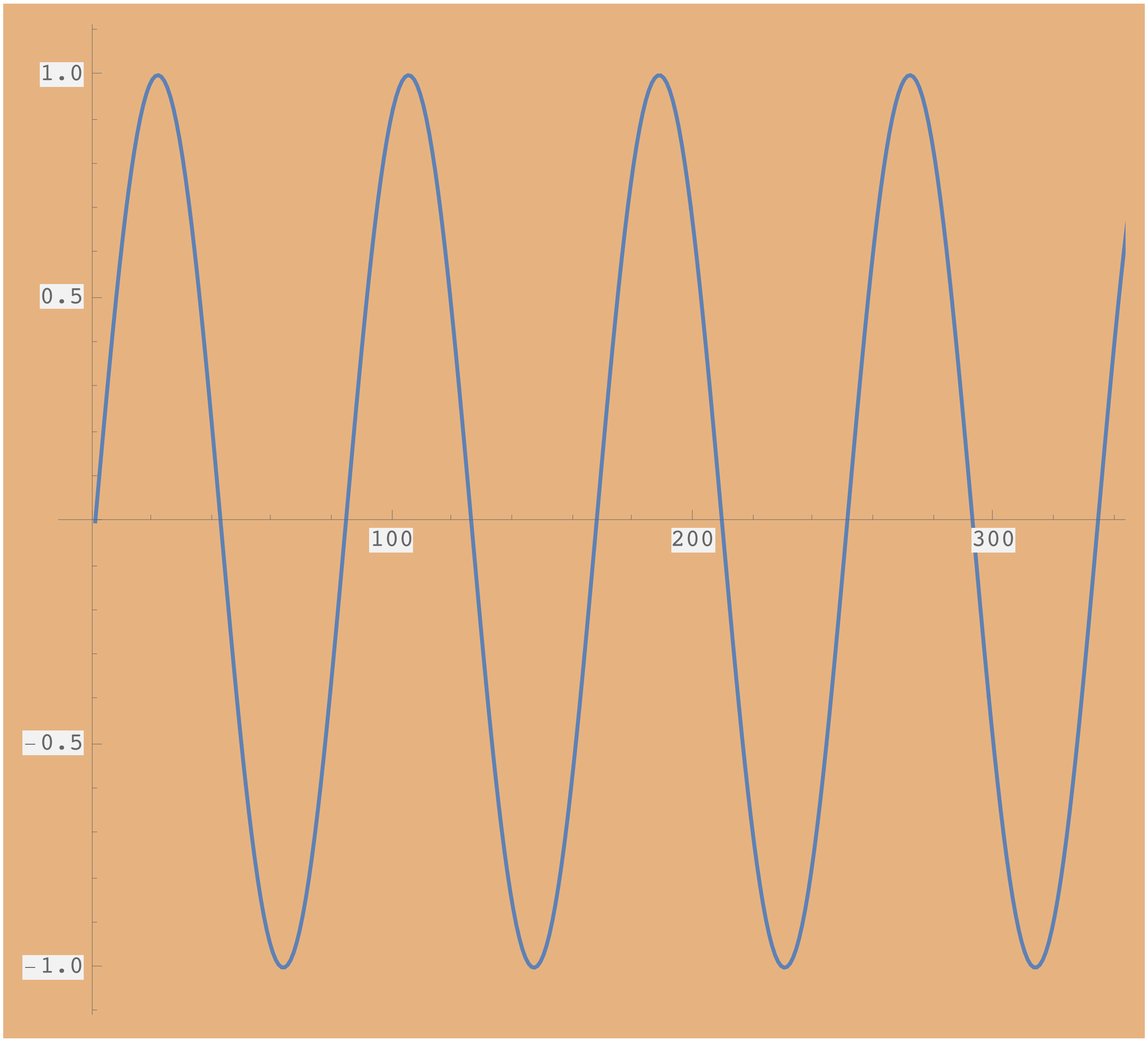
```
n = 512; f = 1.2; fs = 100;
```

```
IndexToFreq[k_, fs_, n_] := k/n fs;
```

The digitized sine wave is

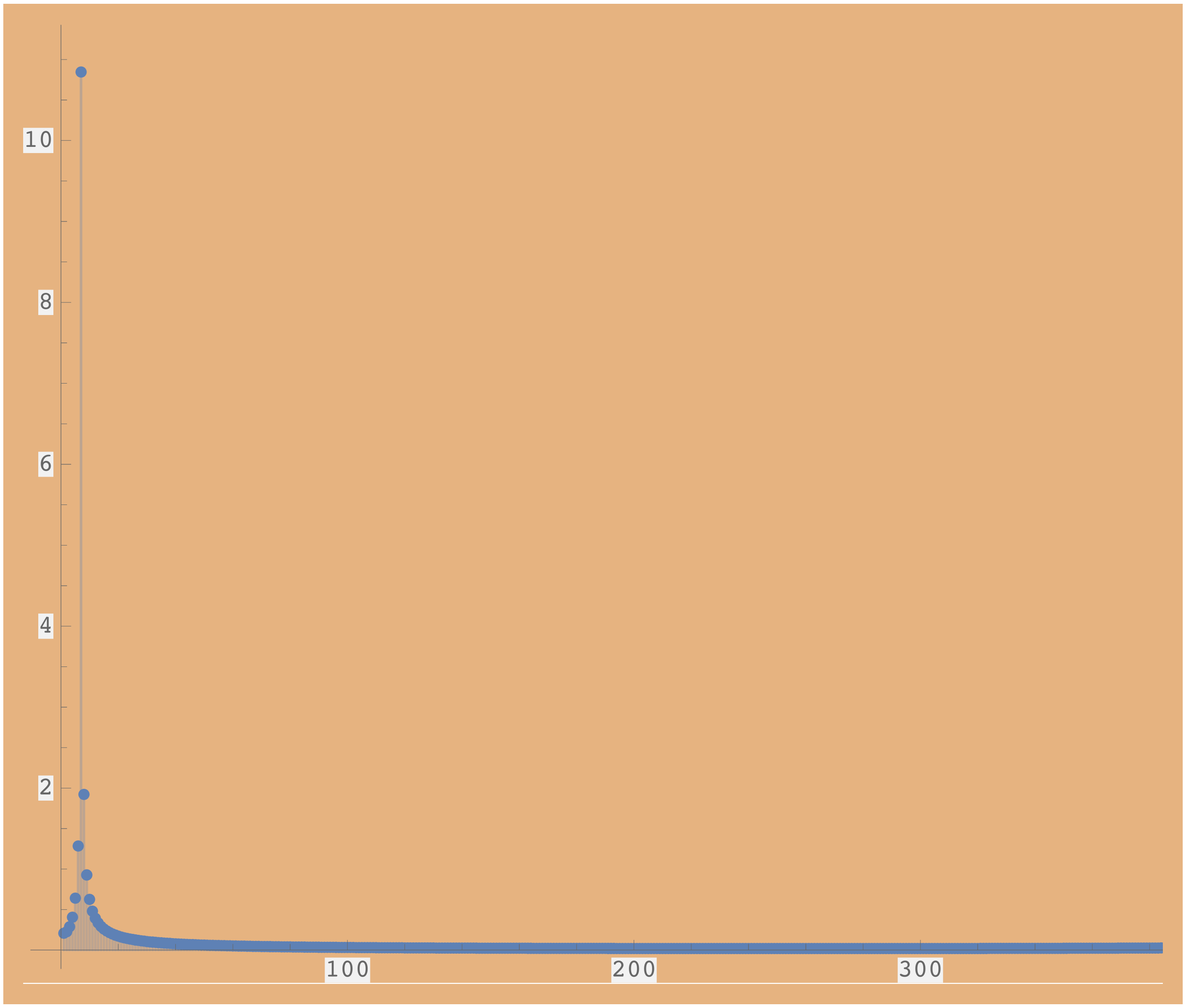
```
DigitalSine[f_, fs_, n_] := Table[Sin[(2 π f)/fs k], {k, 0, n - 1}];
```

```
ListPlot[DigitalSine[f, fs, n],
 Joined -> True, Filling → None, PlotRange → Full]
```



Its power spectrum instead of showing peaks, shows a bit of spreading since there are discontinuities at the endpoints generating high frequencies.

```
ListPlot[Abs[Fourier[DigitalSine[f, fs, n]]],
 Filling → Axis, PlotRange → Full]
```



Mapping the discrete index to the frequency in Hz,

```
IndexToFreq[6, fs, n] // N
```

```
1.17188
```

The maximum frequency is 50 Hz from the sampling theorem,

```
IndexToFreq[n / 2, fs, n] // N
```

```
50.
```

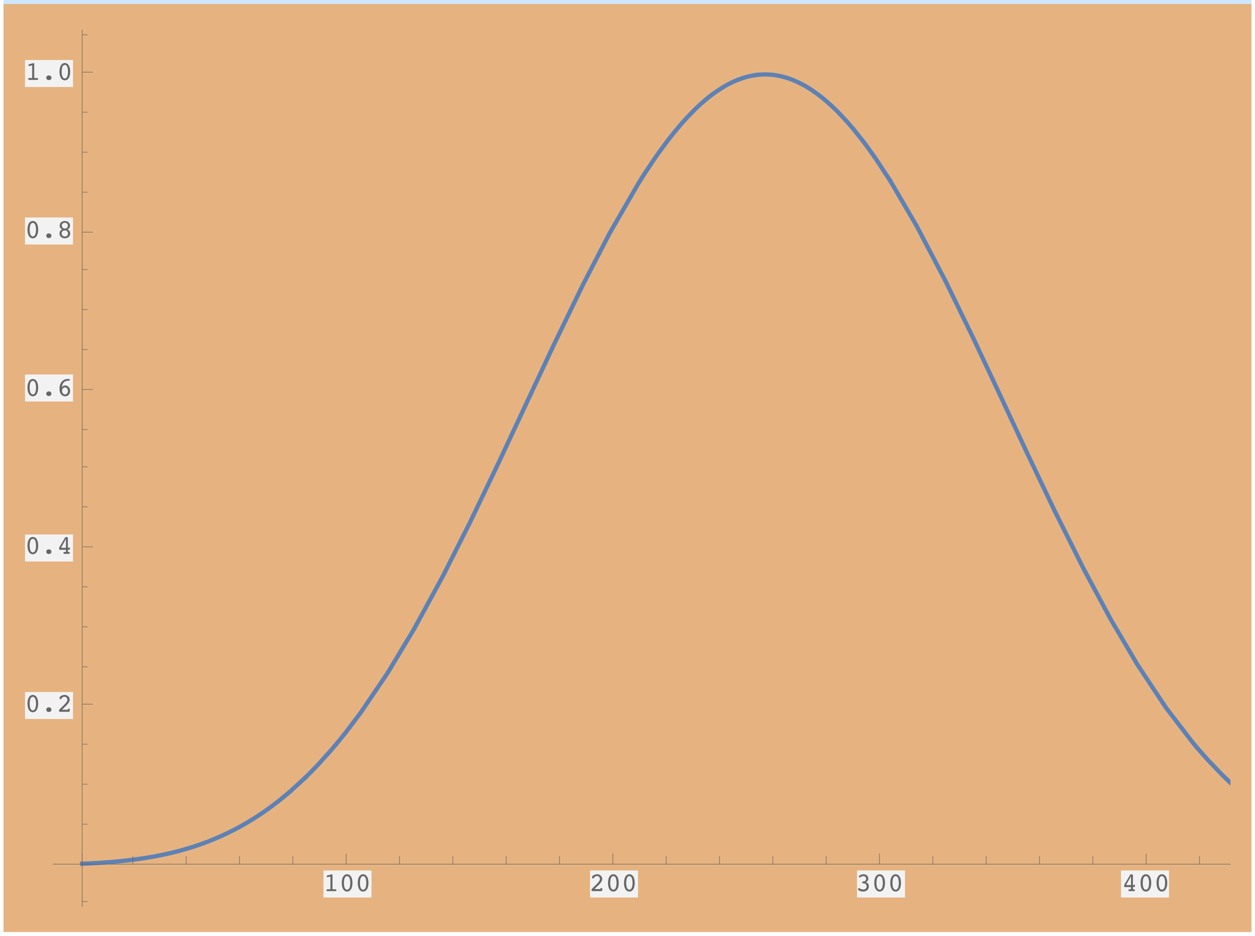
A Kaiser-Bessel window is given by the function,

```
β = 9;
```

```
KaiserBessel[k_, β_] := BesselI[0, β Sqrt[1 - (2 (k-1)/(n-1) - 1)^2]] / BesselI[0, β]
```

$$\text{KaiserBessel}[k\_, \beta\_] := \frac{\text{BesselI}\left[0, \beta \sqrt{1 - \left(2 \frac{k-1}{n-1} - 1\right)^2}\right]}{\text{BesselI}[0, \beta]}$$

```
Plot[KaiserBessel[k, 9], {k, 0, n}]
```

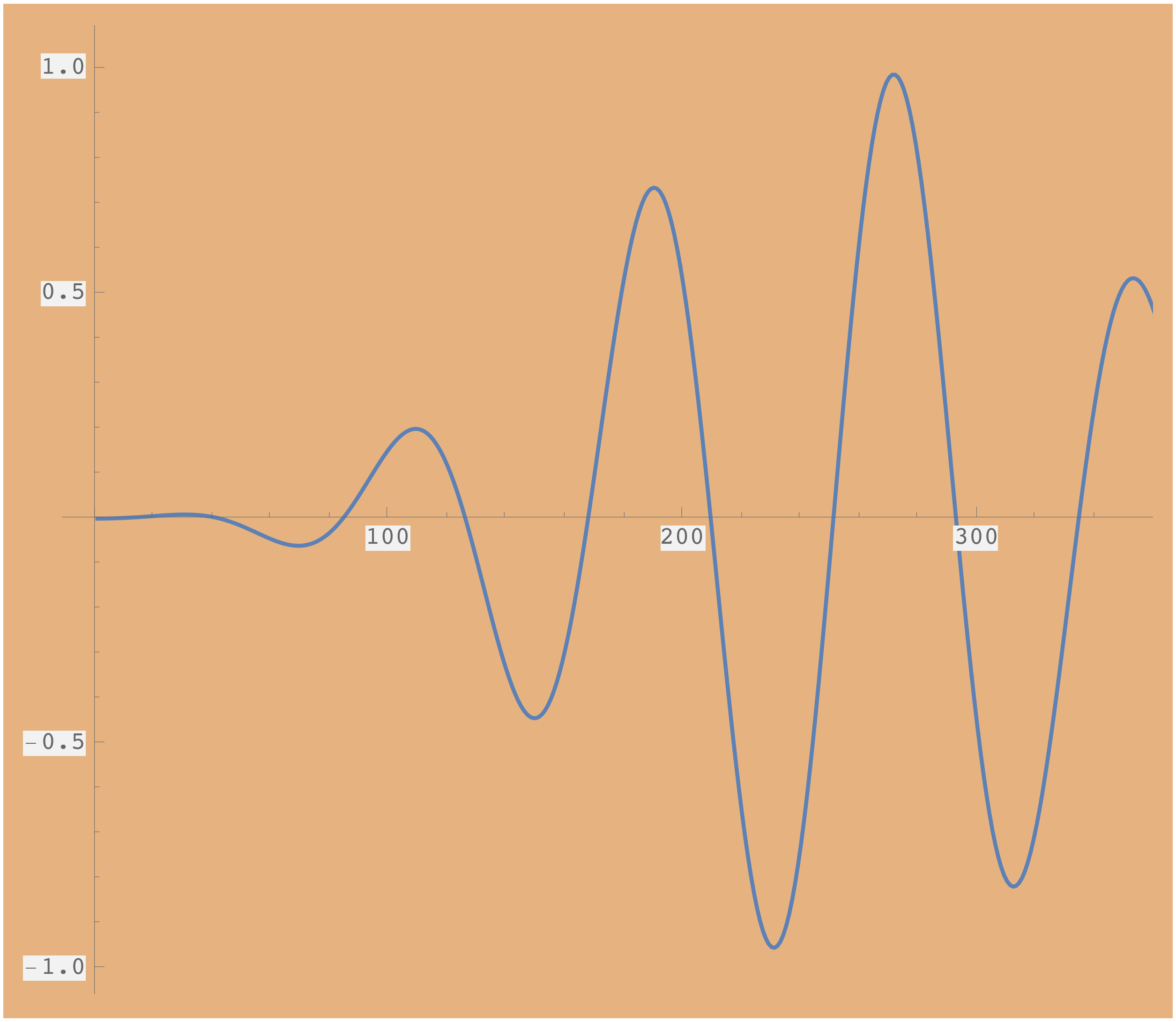


Digitize the Kaiser-Bessel window,

```
KaiserBessel[n_] := Table[KaiserBessel[k, β], {k, 0, n - 1}];
```

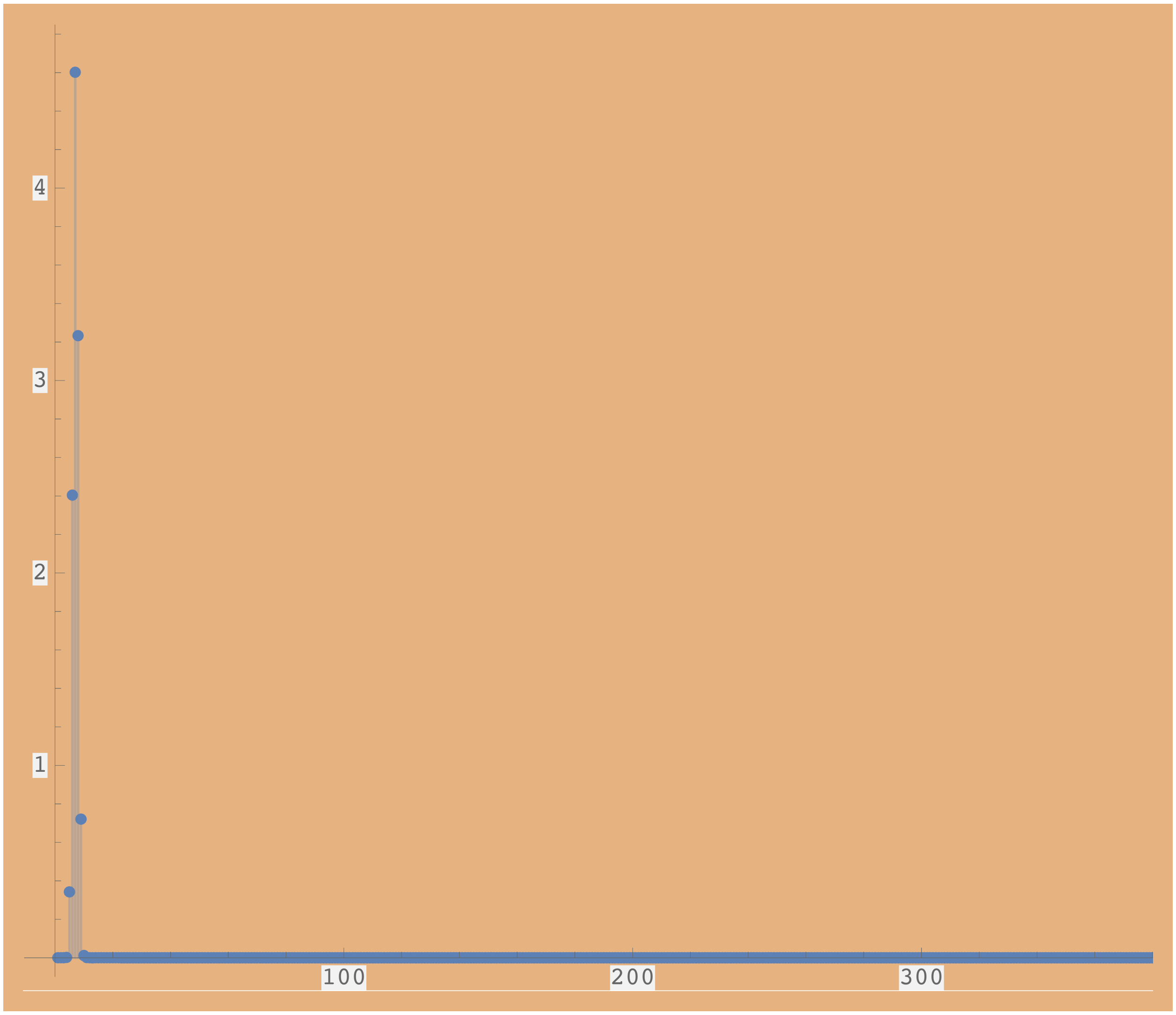
Apply the Kaiser-Bessel window to the sine wave to make it periodic,

```
ListPlot[KaiserBessel[n] DigitalSine[f, fs, n],
 Joined -> True, Filling → None, PlotRange → Full]
```



The power spectrum now has less contamination by high frequency generated discontinuities,

```
ListPlot[Abs[Fourier[DigitalSine[f, fs, n] KaiserBessel[n]]],
 Filling → Axis, PlotRange → Full]
```


4
3
2
1
100
200
300